



Modified quick-switch-based sampling inspection system with Six-Sigma yield considerations for economically filtering unreliable suppliers

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Received: 28 April 2021 / Accepted: 10 October 2021 / Published online: 22 January 2022
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Abstract

The variables quick-switch sampling (VQSS) systems operate by alternating between normal and tightened sampling plans; intrinsically, they efficiently employ the preceding inspection result to provide a dynamic lot disposition mechanism for validating quality capability. So far, most studies of the VQSS systems have focused on the system that applies adjustable acceptance criteria, and changeover of the required sample sizes has rarely been discussed. However, from a managerial perspective, these two conventional types of VQSS systems have different advantages in lot-acceptance sampling practice. To accommodate the sampling strengths of both systems, a modified VQSS (MVQSS) system appraised by the process yield index was developed in this paper. The proposed MVQSS system can alter both the acceptance criterion and the sample size required for tightening lot disposition to obtain superior inspection performance. Meanwhile, practitioners can transform the MVQSS system into two conventional systems when specific conditions are specified; that is, the MVQSS system can be perceived as a generalized VQSS system. On the report of a series of performance comparisons and investigations, we tabulated and summarized the three systems' strengths for some managerial suggestions and proposed a lot-inspection progression process for different stages of the supplier-buyer trust relationship. The results convey clearly that only the suppliers with reliable and Six-Sigma yield submission can benefit from reducing inspection costs. Furthermore, to enhance the proposed systems' practicability, we introduced a web-based app for distribution professionals to create their optimal systems' design online. Finally, a real-world case was demonstrated to illustrate the practical applicability of the proposed system.

Keywords Average run length · Modified quick-switch sampling · Nonlinear optimization · Six-Sigma process yield · Variables sampling plans

1 Introduction

Followed by consumers' continuously stricter requests for reliable products, the delivery of high-quality products has become a vital factor across business sectors for their developmental sustainability [1, 2]. Acceptance sampling plans (ASPs) play a significant role in validating product quality for numerous industrial distributions today [3]. For instance, in the manufacturing and service industries, a produced components' key quality characteristics are generally required to be inspected before being delivered to the next process [4]. Usually, distribution professionals operate attributes-type ASPs to inspect products' quality characteristics to see whether or not they are within the quality specification limit [5]. In the attributes-type inspection, a product's quality characteristic that falls within the quality

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specification limits is perceived as the conforming item regardless of their deviation from the target; in contrast, a product's quality characteristic that deviates more or less from the quality specification limits is all perceived as the nonconforming item. However, as consumers' consciousness has rapidly advanced that even a small variation from a product's performance impacts their satisfaction and loyalty, the acceptance of nonconforming items has decreased to a few parts per million [6]. As a consequence, the traditional attributes-type ASPs that usually demand inspecting large-amount samples are becoming insufficient when validating products with Six-Sigma quality levels in practice [7].

Therefore, the variables-type ASPs gauging the exact measurements of the product's quality characteristics, which can provide more precise quality information than attributes-type ASPs, receive greater attention [8]. Among the variables-type ASPs, several sampling schemes with different lot disposition mechanisms, such as the lot-resampling, the lot-backtracking, and the sampling-rule switching mechanism, have been introduced [9–11]. The variables quick-switch sampling (VQSS) system that can provide a dynamic lot sentencing mechanism based on the previous inspection result is among the efficient quality validation schemes [12]. In most VQSS systems, the lot disposition mechanism switches between two levels of variables single sampling plan (VSSP), i.e., a normal VSSP (NVSSP), and a tightened VSSP (TVSSP). According to the design of the TVSSP, two subdivisions of the VQSS system have been proposed: (i) the VQSS-I system, which exerts a larger required sample size for the TVSSP, and (ii) the VQSS-II system, which includes a more stringent acceptance criterion in the TVSSP.

Recently, the VQSS system has been integrated with the process capability indices (PCIs) to accommodate quality characteristic specifications stipulated by buyers for the lot disposition (see examples in [13–17]). Regarding a normally distributed quality characteristic with bilateral specification limits, the VQSS system in the quantification of the process-yield PCI was introduced by Liu and Wu [18] and Wu et al. [15, 17], the process-loss PCI was established by Balamurali and Usha [13, 19], and the jointly process yield-and-loss PCI was developed by Balamurali and Usha [14]. Concerning a unilateral quality characteristic, Wu et al. [16] and Shu et al. [20] respectively conducted the PCI-based VQSS system under normal and Weibull distributions (2021).

Most of the abovementioned studies focused on the VQSS-II system; Wu et al. [15, 17] had developed and investigated both the VQSS-I and VQSS-II systems. Although the VQSS-I system requires a larger required sample size in the TVSSP that may cause a relatively large average sample size, its sensitivity to quality degradation of the lot is superior, which can offer better protection for buyers than the VQSS-II system does. In other words, from a managerial viewpoint, the VQSS-I and VQSS-II systems have different advantages

in practice. Consequently, the development of the VQSS-I system becomes necessary for offering distribution professionals another option to deal with different situations in the supply chain channel. Moreover, the VQSS-I and VQSS-II systems only account for increasing either the required sample size or the acceptance criterion for lot disposition in the TVSSP. A topic that adopts a higher required sample size and acceptance criterion simultaneously in the TVSSP for lot disposition has yet to be considered in any other study, thus making it an appealing approach.

Hence, a modified VQSS (MVQSS) system appraised by the process yield index was developed in this paper; this MVQSS system does not only operate both larger sample size and acceptance criterion required in the TVSSP to obtain superior performance but can also be transformed into the VQSS-I system or the existing VQSS-II system. That is, the MVQSS system can be perceived as a generalized VQSS system. Moreover, existing studies on the VQSS systems only provide specific tables that cannot cover all combinations of the regulations bound in real-world purchasing contracts for practitioners to execute their proposed systems, which may limit their practicability. To transcend this limitation, we developed a web-based app for practitioners to calculate online for our proposed optimal system design [21]. As a consequence, the practitioners can freely switch between these three types of VQSS systems, namely the VQSS-I system, the VQSS-II system, and the MVQSS system, according to different focusing considerations for lot disposition. This improvement will significantly enhance the practicability of the VQSS systems.

2 Process yield index

In recent decades, PCIs, which have been used to gauge manufacturing process performance concerning buyers' stipulated specifications in statistical quality control, have been a research focus for process capability studies [6]. Process yield is among the critical process performances in practice [1, 2]. Suppose that products' quality characteristic X follows a normal distribution, $X \sim N(\mu, \sigma^2)$, with bilateral specifications, namely, LSL (lower specification limit) and USL (upper specification limit), the process yield can be acquired as

$$\text{Yield} = \Phi\left(\frac{USL - \mu}{\sigma}\right) - \Phi\left(\frac{\mu - LSL}{\sigma}\right) \quad (1)$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function (CDF).

In pioneer studies on PCIs, the precision index C_p and the accuracy index C_a , two basic indices, were investigated to determine how well the supplier's process range is to the

buyer’s specification range and how close the supplier’s process average is to the buyer’s specification average, respectively [22, 23].

$$C_p = \frac{USL - LSL}{6\sigma} \text{ and } C_a = 1 - \frac{|\mu - M|}{d} \tag{2}$$

where $d = (USL - LSL)/2$ and $M = (USL + LSL)/2$ are the half-range and the average of the specification tolerances, respectively.

To provide a precise measure of the actual process yield from Eq. (1) and accommodate both strengths of C_p and C_a from Eq. (3), Boyles [24] introduced the process yield index, signified as S_{pk} , as follows.

$$S_{pk} = \frac{1}{3}\Phi^{-1} \left[\frac{1}{2}\Phi\left(\frac{USL - \mu}{\sigma}\right) + \frac{1}{2}\Phi\left(\frac{\mu - LSL}{\sigma}\right) \right] \tag{3}$$

$$= \frac{1}{3}\Phi^{-1} \left[\frac{1}{2}\Phi(3C_p C_a) + \frac{1}{2}\Phi(3C_p(2 - C_a)) \right]$$

Each S_{pk} value can communicate interchangeable technical terms of the process yield and the process-nonconforming products in parts per million (PPM), denoted as p .

$$\text{Yield} = [2\Phi(3S_{pk}) - 1] \tag{4}$$

$$p = 2[1 - \Phi(3S_{pk})] \times 10^6 \tag{5}$$

Since in Eqs. (1)–(3), the process parameters μ and σ are unknown in practice, by respectively substituting the sample mean \bar{X} and the sample standard deviation S , Lee et al. [25] introduced the natural estimator of S_{pk} , \hat{S}_{pk} , and derived its asymptotic distribution to provide an approximation to the \hat{S}_{pk} ’s CDF:

$$\hat{S}_{pk} = \frac{1}{3}\Phi^{-1} \left[\frac{1}{2}\Phi\left(\frac{USL - \bar{X}}{S}\right) + \frac{1}{2}\Phi\left(\frac{\bar{X} - LSL}{S}\right) \right] \tag{6}$$

$$\hat{S}_{pk} \rightarrow^a N\left(S_{pk}, \frac{v^2 + v^2}{36n \cdot \phi(3S_{pk})^2}\right) \tag{7}$$

where $\phi(\cdot)$ is the probability density function of $N(0, 1)$; v and v are

$$v = \frac{1}{\sqrt{2}} \left[\frac{USL - \mu}{\sigma} \cdot \phi\left(\frac{USL - \mu}{\sigma}\right) + \frac{\mu - LSL}{\sigma} \cdot \phi\left(\frac{\mu - LSL}{\sigma}\right) \right]$$

$$= \frac{1}{\sqrt{2}} [3C_p C_a \phi(3C_p C_a) + 3C_p(2 - C_a)\phi(3C_p(2 - C_a))]$$

$$v = \phi\left(\frac{USL - \mu}{\sigma}\right) - \phi\left(\frac{\mu - LSL}{\sigma}\right)$$

$$= \phi(3C_p(2 - C_a)) - \phi(3C_p C_a)$$

Therefore, the explicit CDF of \hat{S}_{pk} can be written as follows.

$$\Pr(\hat{S}_{pk} \leq x) = \sqrt{\frac{18n}{\pi}} \frac{\phi(3S_{pk})}{\sqrt{v^2 + v^2}} \int_0^x \exp\left[-\frac{18n(\phi(3S_{pk}))^2(t - S_{pk})^2}{v^2 + v^2}\right] dt \tag{8}$$

Notably, parameters v and v are functions of two unknown indices C_p and C_a . To eliminate the persistent demand on estimating C_p and C_a , the setting $C_a = 1$ for an obtained conservative S_{pk} ’s lower confidence bound under a certain confidence level and a testing power was used to guarantee that decision-making is reliable when assessing process-yield capability [15, 17].

Moreover, by substituting Eq. (5) into Eq. (8), the explicit CDF of \hat{S}_{pk} can be rewritten based on process defectives in PPM, p .

$$\Pr(\hat{S}_{pk} \leq x) = \sqrt{\frac{18n}{\pi}} \frac{\phi\left[\Phi^{-1}\left(1 - \frac{p}{2 \times 10^6}\right)\right]}{\sqrt{v^2 + v^2}} \times \int_0^x \exp\left\{-\frac{18n\left\{\phi\left[\Phi^{-1}\left(1 - \frac{p}{2 \times 10^6}\right)\right]\right\}^2 \left\{t - \left[\frac{1}{3}\Phi^{-1}\left(1 - \frac{p}{2 \times 10^6}\right)\right]\right\}^2}{v^2 + v^2}\right\} dt \tag{9}$$

3 Development of modified yield-based quick-switch sampling systems

In this section, the S_{pk} -based MVQSS system (abbr. S_{pk} -MVQSS system) is developed. Because the -MVQSS system is a modified extension of the existing S_{pk} -VQSS system, we will review S_{pk} -VQSS system briefly before designing the S_{pk} -MVQSS system.

3.1 Existing yield-based quick-switch sampling systems

The existing S_{pk} -VQSS system with parameters (n, k_N, k_T) for lot disposition was introduced by Liu and Wu [18], where n is the sample size required for inspection, and k_N and k_T , $k_N < k_T$, are the lot-accepted criteria respectively for NVSSP and TVSSP. Its operational steps proceed as follows:

- Step 1: At onset with NVSSP, draw n items randomly from the submitted lot, and measure their key quality characteristic, namely, X_1, X_2, \dots, X_n , to calculate the \hat{S}_{pk} .
- Step 2: Accept the submitted lot if $\hat{S}_{pk} \geq k_N$ and continue applying the NVSSP; otherwise, reject the submitted lot and move forward to Step 3 for the next submitted lot.
- Step 3: Switch to the TVSSP; draw n items randomly from the submitted lot, and measure their key quality characteristic to calculate the \hat{S}_{pk} .

Step 4: Accept the submitted lot if $\hat{S}_{pk} \geq k_T$ and switch back to Step 1, i.e., NVSSP, for the next submitted lot; otherwise, reject the submitted lot and continue applying in TVSSP.

According to the mentioned operational procedures, the submitted lot could be accepted in Step 2 or Step 4. The accepted probability of a submitted lot in Step 2, symbolized as $P_N^{VQSS}(n, k_N, k_T|p)$, can be derived by referring to Eq. (9) as follows.

$$\begin{aligned}
 P_N^{VQSS}(n, k_N, k_T|p) &= \Pr(\hat{S}_{pk} > k_N|p) \\
 &= \sqrt{\frac{18n}{\pi}} \frac{\phi\left[\Phi^{-1}\left(1 - \frac{p}{2 \times 10^6}\right)\right]}{\sqrt{v^2 + v^2}} \times \\
 &\int_{k_N}^{\infty} \exp\left\{-\frac{18n\left\{\phi\left[\Phi^{-1}\left(1 - \frac{p}{2 \times 10^6}\right)\right]\right\}^2 \left\{t - \left[\frac{1}{3}\Phi^{-1}\left(1 - \frac{p}{2 \times 10^6}\right)\right]\right\}^2}{v^2 + v^2}\right\} dt
 \end{aligned} \tag{10}$$

Likewise, the submitted lot accepted probability in Step 4, denoted as $P_T^{VQSS}(n, k_N, k_T|p)$, can be derived as follows.

$$\begin{aligned}
 P_T^{VQSS}(n, k_N, k_T|p) &= \Pr(\hat{S}_{pk} > k_T|p) \\
 &= \sqrt{\frac{18n}{\pi}} \frac{\phi\left[\Phi^{-1}\left(1 - \frac{p}{2 \times 10^6}\right)\right]}{\sqrt{v^2 + v^2}} \times \\
 &\int_{k_T}^{\infty} \exp\left\{-\frac{18n\left\{\phi\left[\Phi^{-1}\left(1 - \frac{p}{2 \times 10^6}\right)\right]\right\}^2 \left\{t - \left[\frac{1}{3}\Phi^{-1}\left(1 - \frac{p}{2 \times 10^6}\right)\right]\right\}^2}{v^2 + v^2}\right\} dt
 \end{aligned} \tag{11}$$

Accordingly, by referring to Romboski [26] and Eqs. (10) and (11), the overall accepted probability of the submitted lot, $\pi_a^{VQSS}(n, k_N, k_T|p)$, can be obtained as

$$\pi_a^{VQSS}(n, k_N, k_T|p) = \frac{P_T^{VQSS}(n, k_N, k_T|p)}{1 - P_N^{VQSS}(n, k_N, k_T|p) + P_T^{VQSS}(n, k_N, k_T|p)} \tag{12}$$

Subsequently, the optimal S_{pk} -VQSS system is designed to satisfy two critical points, $(p_{ANL}, 1 - \alpha)$ and (p_{RNL}, β) , on the operating characteristic (OC) curve, where p_{ANL} and p_{RNL} are the acceptable nonconforming and the rejectable nonconforming levels of p , defined in Eq. (5), respectively; α and β are the producer’s and consumer’s risks, respectively; that is to say when process yield is at p_{ANL} , the submitted lot would be accepted at least $(1 - \alpha)\%$, and at p_{RNL} , it would be accepted at most $\beta\%$. The following two equations can express these two constraints:

$$\begin{cases} \Pr(\text{accepting the submitted lot}|p_{ANL}) \geq 1 - \alpha; \\ \Pr(\text{accepting the submitted lot}|p_{RNL}) \leq \beta. \end{cases} \tag{13}$$

Finally, by employing Eq. (13), an optimization model can be established to determine the optimal system design (n, k_N, k_T) based on the economic consideration of minimizing the sample size required for inspection, as follows.

$$\begin{aligned}
 &\text{Min } [n'] = n \\
 &\text{Subject to} \\
 &\pi_a^{VQSS}(n', k_N, k_T|p_{ANL}) \geq 1 - \alpha \\
 &\pi_a^{VQSS}(n', k_N, k_T|p_{RNL}) \leq \beta \\
 &1 < n'; k_N < k_T
 \end{aligned} \tag{14}$$

where $[n']$ is the smallest integer greater than or equal to n' .

Notably, after reviewing Liu and Wu [18], we discovered that the existing S_{pk} -VQSS system only discussed the S_{pk} -VQSS- (n, k_N, k_T) system, i.e., the S_{pk} -VQSS-II system. Another S_{pk} -VQSS- (n_N, n_T, k) system, i.e., the S_{pk} -VQSS-I system, has not been addressed in the literature so far, and will be included in the next section.

3.2 Modified yield-based quick-switch sampling system

Unlike the existing S_{pk} -VQSS system, our proposed S_{pk} -MVQSS system operates four parameters: (n_N, n_T, k_N, k_T) , for lot disposition, where n_N and n_T , $n_N < n_T$, are the required sample size in the NVSSP and TVSSP, respectively. Thus, the S_{pk} -MVQSS system can adopt both larger required sample size and a more stringent lot-accepted criterion, i.e., (n_T, k_T) , in TVSSP to sentence the submitted lot. The S_{pk} -MVQSS system’s operational steps proceed as follows:

- Step 1: Start with NVSSP; draw n_N items randomly from the submitted lot and measure their key quality characteristic to calculate the \hat{S}_{pk} .
- Step 2: Accept the submitted lot if $\hat{S}_{pk} \geq k_N$ and remain in NVSSP; otherwise, reject the submitted lot and move forward to Step 3 for the next submitted lot.
- Step 3: Switch to TVSSP; draw n_T items randomly from the submitted lot and measure their key quality characteristic to calculate the \hat{S}_{pk} .
- Step 4: Accept the submitted lot if $\hat{S}_{pk} \geq k_T$, and switch back to Step 1, i.e., NVSSP, for the next submitted lot; otherwise, reject the submitted lot and remain in TVSSP.

Likewise, the submitted lot will be accepted in Step 2 or Step 4. The acceptance probability of the submitted lot in Steps 2 and 4, i.e., $P_N^{MVQSS}(n_N, n_T, k_N, k_T|p)$ and $P_T^{MVQSS}(n_N, n_T, k_N, k_T|p)$, can be derived as follows.

$$\begin{aligned}
 P_N^{MVQSS}(n_N, n_T, k_N, k_T|p) &= \Pr(\hat{S}_{pk} > k_N|p) \\
 &= \sqrt{\frac{18n_N}{\pi}} \frac{\phi\left[\Phi^{-1}\left(1 - \frac{p}{2 \times 10^6}\right)\right]}{\sqrt{v^2 + v^2}} \times \\
 &\int_{k_N}^{\infty} \exp\left\{-\frac{18n_N\left\{\phi\left[\Phi^{-1}\left(1 - \frac{p}{2 \times 10^6}\right)\right]\right\}^2 \left\{t - \left[\frac{1}{3}\Phi^{-1}\left(1 - \frac{p}{2 \times 10^6}\right)\right]\right\}^2}{v^2 + v^2}\right\} dt
 \end{aligned} \tag{15}$$

$$\begin{aligned}
 P_T^{MVQSS}(n_N, n_T, k_N, k_T | p) &= \Pr(\hat{S}_{pk} > k_T | p) \\
 &= \sqrt{\frac{18n_T}{\pi}} \frac{\phi\left[\Phi^{-1}\left(1 - \frac{p}{2 \times 10^6}\right)\right]}{\sqrt{v^2 + v'^2}} \times \\
 &\int_{k_T}^{\infty} \exp\left\{-\frac{18n_T\left\{\phi\left[\Phi^{-1}\left(1 - \frac{p}{2 \times 10^6}\right)\right]\right\}^2\left\{t - \left[\frac{1}{3}\Phi^{-1}\left(1 - \frac{p}{2 \times 10^6}\right)\right]\right\}^2}{v^2 + v'^2}\right\} dt
 \end{aligned}
 \tag{16}$$

Subsequently, the S_{pk} -MVQSS system’s overall acceptance probability of the submitted lot, denoted as $\pi_a^{MVQSS}(n_N, n_T, k_N, k_T | p)$, can be obtained as

$$\pi_a^{MVQSS}(n_N, n_T, k_N, k_T | p) = \frac{P_T^{MVQSS}(n_N, n_T, k_N, k_T | p)}{1 - P_N^{MVQSS}(n_N, n_T, k_N, k_T | p) + P_T^{MVQSS}(n_N, n_T, k_N, k_T | p)}
 \tag{17}$$

Moreover, according to the operational procedures, the sample size required for inspection in S_{pk} -MVQSS system is switched between n_N and n_T . Thus, we considered employing the average sample size (ASN) at the acceptable nonconforming level p_{ANL} , i.e., $ASN(p_{ANL})$, to be the objective of the optimization model [27]. According to the theories done in Govindaraju and Kuralmani [28], the ASN function of the S_{pk} -MVQSS system can be obtained as follows.

$$\begin{aligned}
 ASN(n_N, n_T, k_N, k_T | p_{ANL}) &= \\
 &= \frac{P_T^{MVQSS}(n_N, n_T, k_N, k_T | p_{ANL}) \cdot n_N + \left[1 - P_N^{MVQSS}(n_N, n_T, k_N, k_T | p_{ANL})\right] \cdot n_T}{1 - P_N^{MVQSS}(n_N, n_T, k_N, k_T | p_{ANL}) + P_T^{MVQSS}(n_N, n_T, k_N, k_T | p_{ANL})}
 \end{aligned}
 \tag{18}$$

Consequently, to determine the optimal system design (n_N, n_T, k_N, k_T) , the optimization model of S_{pk} -MVQSS system can be established by referring to Eqs. (13), (17), and (18) as follows.

$$\begin{aligned}
 \text{Min}_{n_N, n_T, k_N, k_T} \quad &ASN(n_N, n_T, k_N, k_T | p_{ANL}) \\
 \text{Subject to} \quad & \\
 \pi_a^{MVQSS}(n_N, n_T, k_N, k_T | p_{ANL}) &\geq 1 - \alpha \\
 \pi_a^{MVQSS}(n_N, n_T, k_N, k_T | p_{RNL}) &\leq \beta \\
 1 < n_N < n_T; k_N < k_T
 \end{aligned}
 \tag{19}$$

Particularly, the S_{pk} -MVQSS system can be regarded as a consolidated and adjustable sampling scheme because it can convert into other types of S_{pk} -VQSS systems when the succeeding specified conditions are fulfilled: (i) if $k_N = k_T$, then the S_{pk} -MVQSS system reduces to the S_{pk} -VQSS-I system; (ii) if $n_N = n_T$, then the S_{pk} -MVQSS system shrinks to the existing S_{pk} -VQSS-II system, as proposed by Liu and Wu in [18].

4 A web-based app to design optimal quick-switch sampling system

To determine the optimal design of the S_{pk} -MVQSS system, we programmed Eq. (19) in the R software [29]. The optimization package “nloptr” was used for the nonlinearly constrained problem and optimized with minimum ASN [30], where a direct search algorithm was employed [31].

Traditionally, to refrain from solving the complicated nonlinearly constrained optimization problems, scholars provide tables for distribution professionals to implement their

proposed sampling systems. However, tables might only confine the systems in a few regulations, which limit their practical use. Moreover, since our proposed S_{pk} -MVQSS system inherits three different types, the tables-providing method may become excessively burdensome and inconvenient. A flexible manner, therefore, is desirable to allow distribution professionals to obtain an optimal system design that suits their requirements.

In this aim, we adopted the R Shiny package [32] to develop a web-based app to calculate online the S_{pk} -MVQSS system’s optimal design [21]. The web-based app can be quickly accessed by the following hyperlink https://quality-engineering-laboratory.shinyapps.io/spk-mvqss_system_design_calculator/ or by scanning the following quick response (QR) code in Fig. 1.

Firstly, by clicking the items on the left of the user interface (UI), practitioners can freely choose their preferred type of S_{pk} -VQSS system, i.e., the S_{pk} -VQSS-I, S_{pk} -VQSS-II, and S_{pk} -MVQSS systems. Secondly, by importing the specified regulations in the UI of the web-based app we developed, practitioners can obtain an optimal system design speedily and conveniently. For example, if the practitioner adopts the S_{pk} -MVQSS system and the regulations are set to $(p_{ANL}, p_{RNL}, \alpha, \beta) = (100, 1000, 0.05, 0.10)$ in the purchasing contract, the practitioners can obtain the optimal system design $(n_N, n_T, k_N, k_T) = (59, 153, 1.0968, 1.1969)$ through the UI of the web-based app (Fig. 2).

This result demonstrates, at the beginning of the system implementation, that the practitioner should operate the NVSSP- (n_N, k_N) for lot disposition, i.e., draw 59 items from the submitted lot randomly to compute the \hat{S}_{pk} according to these 59 items’ measurements. If $\hat{S}_{pk} \geq k_N$, the submitted lot should be accepted and remain in the S_{pk} -NVSSP- (n_N, k_N) for the succeeding submitted lot; otherwise, the submitted



Fig. 1 QR code of the web-based app

lot would be rejected and switched to the S_{pk} -TVSSP- (n_T, k_T) for the next submitted lot.

Moreover, to reveal the computational efficiency of the app when performing different systems’ designs, namely VQSS-I (n, k_N, k_T) , VQSS-II (n_N, n_T, k) , and MVQSS (n_N, n_T, k_N, k_T) . We simulated the app 1000 times under some widely considered Six-Sigma yields to show computational mean times for each sampling system listed in Table 1. The

Table 1 Mean computing times for each sampling system (unit: second)

$(p_{ANL}, p_{RNL}, \alpha, \beta) = (1, 100, 0.05, 0.05)$		
VQSS-I system	VQSS-II system	MVQSS system
0.138	0.030	0.246
$(p_{ANL}, p_{RNL}, \alpha, \beta) = (100, 1000, 0.05, 0.10)$		
VQSS-I system	VQSS-II system	MVQSS system
0.140	0.032	0.301
$(p_{ANL}, p_{RNL}, \alpha, \beta) = (1, 1000, 0.10, 0.10)$		
VQSS-I system	VQSS-II system	MVQSS system
0.082	0.032	0.234

results are consistent with the difficulty of algorithmic searching for the system’s criteria, where VQSS-II is the quickest, followed by the VQSS-I, and ending with the MVQSS system. Overall, without encountering internet traffic or disruption, the optimum system criteria can be achieved in less than one second. It can be noted that, however, for some uncommon regulations or unsuitable input values, one needs to restart and change the input regulation values when the computing time takes more than 10 seconds.

5 Performance comparisons

In this section, performances of the S_{pk} -VQSS-I, S_{pk} -VQSS-II, and S_{pk} -MVQSS systems were compared in respect of the ASN, the operating characteristic (OC) curves, and the average run length (ARL).

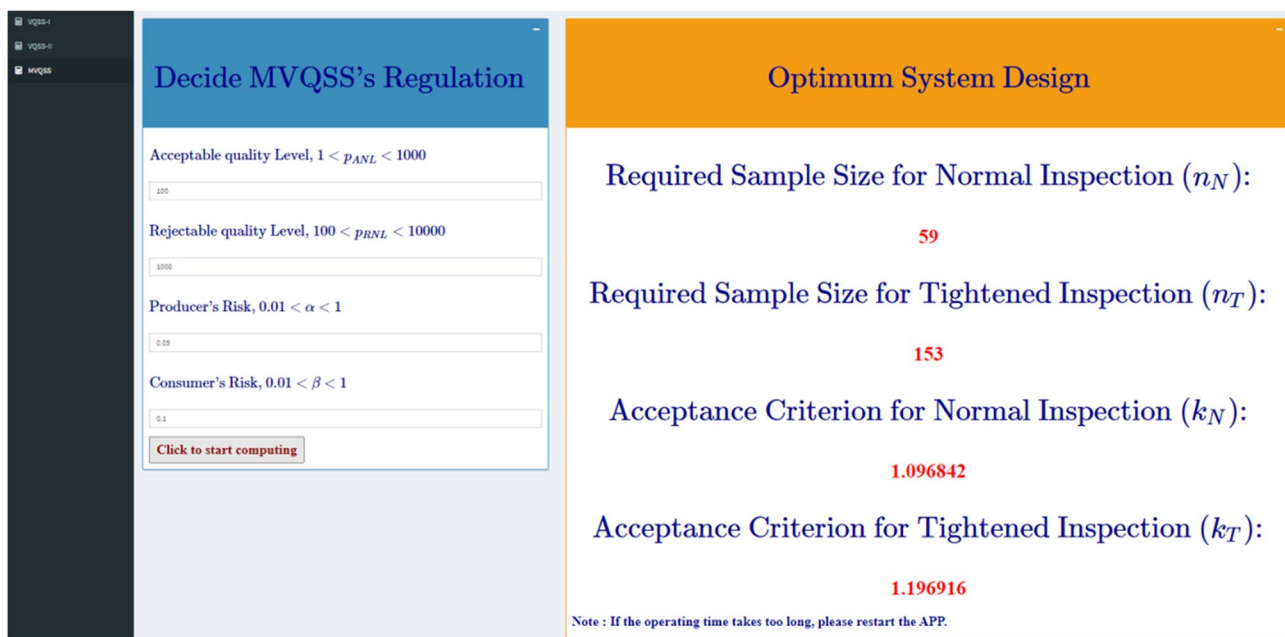


Fig. 2 UI of the web-based app

Table 2 ASN values of the S_{pk} -VQSS-I system, S_{pk} -VQSS-II system, and S_{pk} -MVQSS system under some yield-and-risk regulations ($p_{ANL}, p_{RNL}, \alpha, \beta$)

α	β	$p_{ANL}=1, p_{RNL}=100$		
		S_{pk} -VQSS-I system	S_{pk} -VQSS-II system	S_{pk} -MVQSS system
0.01	0.01	108.02	73	66.18
	0.05	98.68	69	65.71
	0.10	93.43	68	65.48
0.05	0.01	84.93	49	38.26
	0.05	71.44	41	36.42
	0.10	64.15	39	35.52
0.10	0.01	71.38	46	29.38
	0.05	56.51	30	26.29
	0.10	48.80	28	24.81
$p_{ANL}=100, p_{RNL}=1000$				
0.01	0.01	194.08	131	116.74
	0.05	176.76	123	115.84
	0.10	167.02	120	115.40
0.05	0.01	155.21	96	68.27
	0.05	129.90	74	64.71
	0.10	116.28	69	62.99
0.10	0.01	132.02	91	53.25
	0.05	103.88	56	47.28
	0.10	89.34	49	44.41

5.1 Average sample size

The size of the ASN responds to the inspection cost of the system directly. The smaller the size of the ASN, the more cost-efficient the system is. To compare the cost-efficiency

of the S_{pk} -VQSS-I, S_{pk} -VQSS-II, and S_{pk} -MVQSS systems, we tabulated their ASN values under various yield-and-risk regulations ($p_{ANL}, p_{RNL}, \alpha, \beta$) (Table 2) as follows.

Table 2 reveals that the proposed S_{pk} -MVQSS system has the lowest ASN value, whereas the S_{pk} -VQSS-I system has the largest ASN value in all the yield-and-risk regulations. Nevertheless, since the required sample size of S_{pk} -MVQSS system and S_{pk} -VQSS-I system is changed according to the process yield of the submitted lot, we further plot the ASN curves of the S_{pk} -VQSS-I, S_{pk} -VQSS-II, and S_{pk} -MVQSS systems under various process yields by using the yield-and-risk regulations (Fig. 3) as follows.

Figure 3 reveals that the ASN values of the S_{pk} -VQSS-II system remain consistent, whereas the ASN values of the S_{pk} -MVQSS and S_{pk} -VQSS-I systems increase as the process yield decreases. This result reveals that the S_{pk} -MVQSS system and S_{pk} -VQSS-I system will increase the sample size required for inspection when the yield of the submitted lot decreases to safeguard the yield-and-risk regulations for the purchasing contract. When the process yield of the submitted lot is acceptable, i.e., $p \leq p_{ANL}$, the S_{pk} -MVQSS system has the smallest ASN value, which benefits trustworthy suppliers. Conversely, when the process yield of the submitted lot is rejectable, i.e., $p > p_{RNL}$, the ASN values of the S_{pk} -MVQSS and S_{pk} -VQSS-I systems are higher than those of the existing S_{pk} -VQSS-II system. This situation can be seen as punishing the unreliable suppliers in terms of inspection costs.

5.2 Operating characteristic curves

The OC curve manifested the accepted probabilities of a submitted lot against different process yields. Especially at the

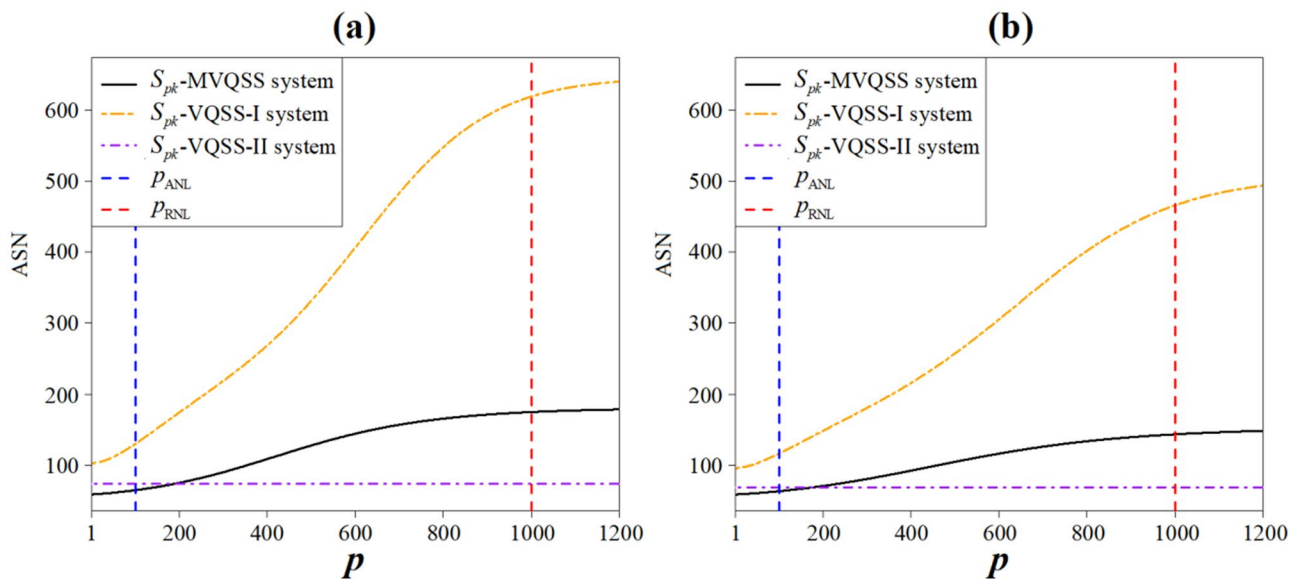
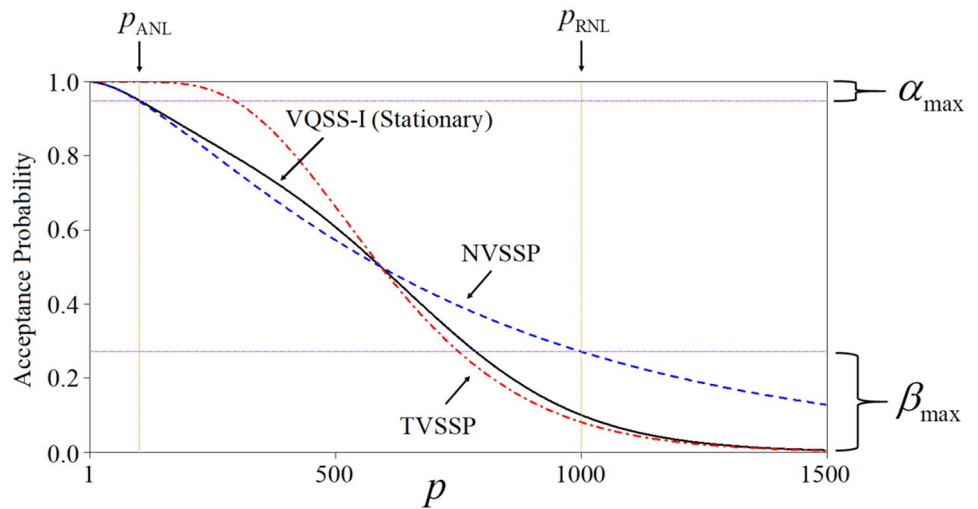


Fig. 3 ASN curves under (a) ($p_{ANL}, p_{RNL}, \alpha, \beta$) = (100, 1000, 0.05, 0.05) and (b) ($p_{ANL}, p_{RNL}, \alpha, \beta$) = (100, 1000, 0.05, 0.10)

Fig. 4 OC curves of the S_{pk} -VQSS-I system



two regulated points, i.e., $(p_{ANL}, 1 - \alpha)$ and (p_{RNL}, β) , practitioners can observe how much of the producer's risk α and how much of the consumer's risk β they should tolerate under p_{ANL} and p_{RNL} , respectively. Regarding the S_{pk} -MVQSS systems, the p_{ANL} and p_{RNL} are determined using the stationary OC curve. In other words, they only consolidate protection during periods of constant process yield. Protection during periods of changing process yield can be described by α_{max} , the maximum producer's risk measured at the p_{ANL} , and β_{max} , the maximum consumer's risk detected at the p_{RNL} , where α_{max} and β_{max} can be calculated by Eqs. (15)–(16).

$$\alpha_{max} = \max \begin{cases} 1 - P_N^{MVQSS}(p_{ANL}) \\ 1 - P_T^{MVQSS}(p_{ANL}) \end{cases} \quad (20)$$

$$\beta_{max} = \max \begin{cases} P_N^{MVQSS}(p_{RNL}) \\ P_T^{MVQSS}(p_{RNL}) \end{cases} \quad (21)$$

Figures 4, 5, to 6 show the S_{pk} -VQSS-I, S_{pk} -VQSS-II, and S_{pk} -MVQSS systems under the yield-and-risk regulations $(p_{ANL}, p_{RNL}, \alpha, \beta) = (100, 1000, 0.05, 0.10)$. We also tabulated the summary statistics of the S_{pk} -VQSS-I, S_{pk} -VQSS-II, and S_{pk} -MVQSS systems (Table 3) as follows.

Figures 4, 5, to 6 and Table 3 reveal that during periods of changing process yield, the S_{pk} -VQSS-II system has the maximum α_{max} and β_{max} , whereas the S_{pk} -VQSS-I system has the lowest α_{max} and β_{max} . These outcomes indicate that the S_{pk} -VQSS-II system should bear the greater risk when process yield changes emerge.

5.3 Average run length

Because the lot disposition of the S_{pk} -VQSS-I, S_{pk} -VQSS-II, and S_{pk} -MVQSS systems is made referring to the preceding quality records, their lot-sentencing sensitivity to process-yield variance in the submitted lots should be considered. In

Fig. 5 OC curves of the S_{pk} -VQSS-II system

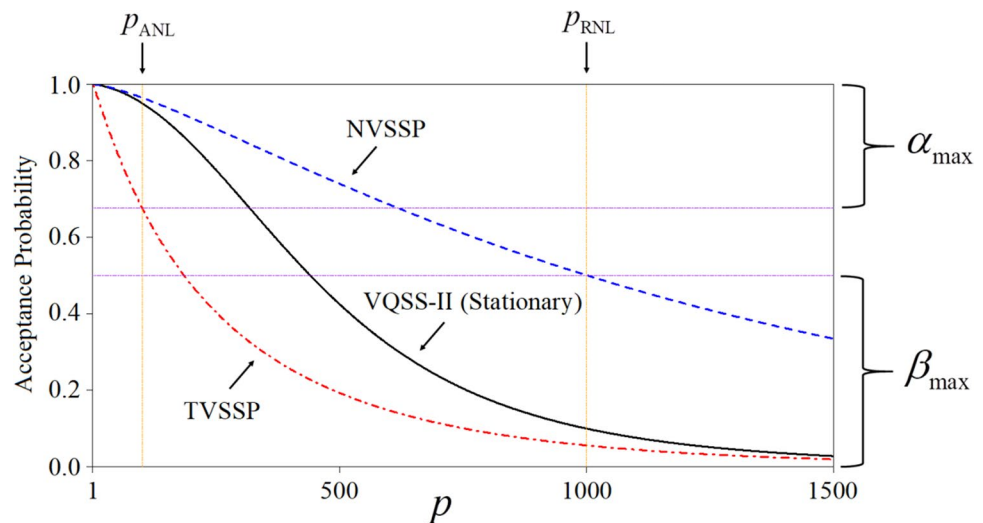
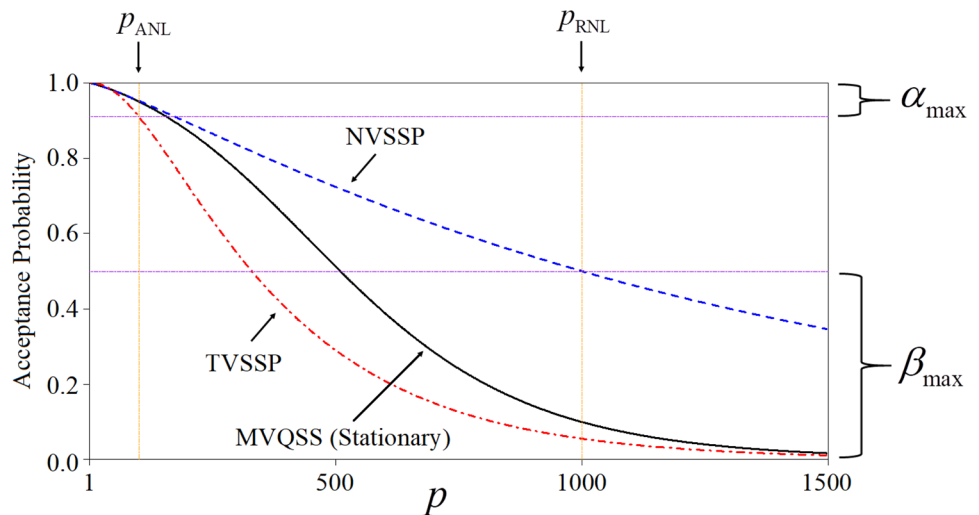


Fig. 6 OC curves of the S_{pk} -MVQSS system



this regard, we adopted the measure of average run length-1 (ARL-1) proposed by Romboski [26].

The ARL - 1 is the average lot numbers counted until the occurrence of rejection at a random lot (counted as the first one) where the process yield changes abruptly to inferior from p_0 to p_1 , $p_1 > p_0$. By referring to Romboski [26], in this paper, the ARL - 1 can be expressed as follows.

$$ARL - 1 = 1 + \frac{[P_N^{MVQSS}(p_0) \times P_N^{MVQSS}(p_1) + P_T^{MVQSS}(p_0) \times P_T^{MVQSS}(p_1)]}{1 - P_N^{MVQSS}(p_1)} \tag{22}$$

As a result, the lower the ARL-1 value of the system is, the greater the lot-sentencing sensitivity of process yield variance is. To compare the sensitivity of the S_{pk} -VQSS-I, S_{pk} -VQSS-II, and S_{pk} -MVQSS systems for the process yield variance in the submitted lots, we tabulated their ARL-1 values using $p_0 = 100$ and various p_1 under the yield-and-risk regulations $(p_{ANL}, p_{RNL}, \alpha, \beta) = (100, 1000, 0.05, 0.10)$ (Table 4) as follows.

Table 4 demonstrates that the S_{pk} -VQSS-I system has the smallest ARL-1 value, which shows that it has the best sensitivity to process yield variance. Conversely, the S_{pk} -VQSS-II system has the largest ARL-1 value, which indicates that it has the lowest sensitivity to process yield variance.

6 A progressive process for lot-inspection applications

Table 5 shows that different systems have different strengths and weaknesses. For example, the S_{pk} -VQSS-I system has the largest ASN value but has the smallest α_{max} , β_{max} and ARL values. These results reveal that the S_{pk} -VQSS-I system has

the lowest cost-efficiency, but also has the least amount of producer-and-consumer risks when process yield varies and is optimally sensitive to it. To integrate these three systems' strengths, we will introduce a progressive application process in the next section and provide managerial suggestions.

Because the S_{pk} -VQSS-I, S_{pk} -VQSS-II and S_{pk} -MVQSS systems have different strengths, we considered their execution for different stages of the supplier-buyer relationship. Initially, we suggested adopting the S_{pk} -VQSS-I system for a new supplier because the supplier had not demonstrated her process yield, yet. The S_{pk} -VQSS-I system has the greatest lot-sentencing sensitivity to process yield variance and the least amount of producer-and-consumer risks, providing a superior mechanism for constructing a trusting relationship between the supplier and buyer.

Table 3 Summary of α_{max} and β_{max} under the regulation $(p_{ANL}, p_{RNL}, \alpha, \beta) = (100, 1000, 0.05, 0.10)$

System type	S_{pk} -VQSS-I system	S_{pk} -VQSS-II system	S_{pk} -MVQSS system
Optimal system design	$(n_N, n_T, k) = (96, 507, 1.1450)$	$(n, k_N, k_T) = (69, 1.0968, 1.2462)$	$(n_N, n_T, k_N, k_T) = (59, 153, 1.0968, 1.1969)$
α_{max}	0.052338	0.323143	0.088769
β_{max}	0.271469	0.500180	0.500167

Table 4 ARL-1 values with $p_0 = p_{ANL} = 100$ and various p_1 under $(p_{ANL}, p_{RNL}, \alpha, \beta) = (100, 1000, 0.05, 0.10)$

p_1	S_{pk} -VQSS-I system	S_{pk} -VQSS-II system	S_{pk} -MVQSS system
105	35.42	44.47	38.28
110	32.97	41.43	36.06
115	30.80	38.73	34.05
120	28.86	36.30	32.22
125	27.13	34.12	30.56
130	25.58	32.15	29.04

Suppose the supplier has consistently earned the buyer’s trust by her Six-Sigma process-yield submissions; the supplier can then be recommended for being transferred to the S_{pk} -VQSS-II system for lot disposition. During this stage, the supplier can benefit from reducing the ASN. However, both the supplier and buyer should note that once the process yield has changed, they must bear the largest α_{max} and β_{max} risks and the lowest sensitivity to process-defectives variance. Therefore, once the supplier has discovered that she cannot ensure that submissions are reliable, we consider restoring the supplier to the previous stage to regain the buyer’s trust.

Finally, if the supplier has demonstrated superior process-yield distributions, in the long run, we suggest employing the S_{pk} -MVQSS system to help the supplier maximize the benefit of reducing the ASN. Notably, during this stage, once the supplier has discovered that she cannot ensure that her submissions are reliable, the supplier should return to the first stage to re-demonstrate the process yield of her submissions and regain the buyer’s trust. This progressive lot-inspection process can be outlined in a simple flowchart (Fig. 7), where the forward and backward directions are respectively represented by solid and dashed lines.

7 A case study

In this section, the proposed S_{pk} -MVQSS system was adopted for a real-world case to demonstrate its practical applicability. The thickness of ultrathin silicon dioxide

Table 5 Summary of performance results from Sects. 5.1 to 5.3

	S_{pk} -VQSS-I system	S_{pk} -VQSS-II system	S_{pk} -MVQSS system
ASN	H	M	S
α_{max}	S	L	M
β_{max}	S	L	M
ARL	S	L	M

L the largest one; *M* the medium one; *S* the smallest one

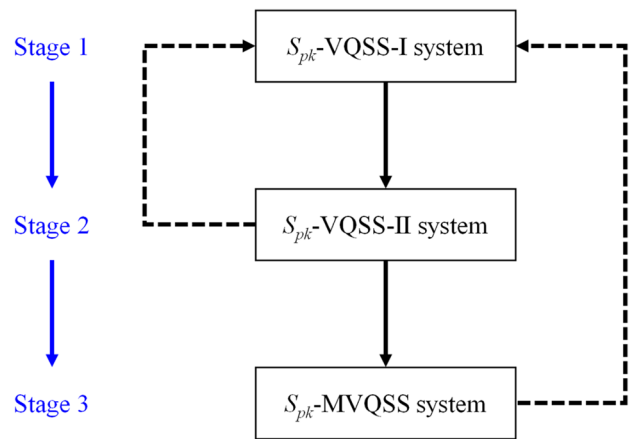


Fig. 7 Flowchart of the introduced progressive process for executing the S_{pk} -VQSS-I system, S_{pk} -VQSS-II system, and S_{pk} -MVQSS system

film (USDF) on a silicon substrate is a critical quality characteristic. Determining such thickness is essential for characterizing the gate oxide growth process of nanoscale metal oxide semiconductor devices. Spectroscopic ellipsometry, a high-precision, efficient metrology for measuring nanofilms, has been widely applied for the last decade. The schematic samples measurement through the spectroscopic ellipsometry and its experimental setup in the lab are shown in Fig. 8a, b.

Suppose the thickness of the USDF for the product is tolerated from 2.5nm to 3.5nm, i.e., the $LSL = 2.5$ nm and the $USL = 3.5$ nm, and the yield-and-risk regulations $(p_{ANL}, p_{RNL}, \alpha, \beta) = (100, 1000, 0.05, 0.10)$ were specified in the supplier-buyer purchasing contract. If the supplier has demonstrated her reliable and Six-Sigma process yield, we suggest employing the S_{pk} -MVQSS system in this case. By operating our developed web-based app, we obtained the optimal system design $(n_N, n_T, k_N, k_T) = (59, 153, 1.0968, 1.1969)$. Following this result, the practitioners randomly sampled 59 items from the submitted lot initially and measured their USDF thickness by spectroscopic ellipsometry. The measurements of these 59 items with their \bar{x} and s and the Anderson–Darling p - value normality test were tabulated in Table 6. Furthermore, its histogram and normal quantile-quantile (Q-Q) plots were profiled in Fig. 9a, b, respectively.

Based on the normality test and normal Q-Q plot, we can conclude that the measurements follow a normal distribution. Subsequently, the estimator of S_{pk} of the submitted lot, \hat{S}_{pk} , was calculated by referring to Eq. (6) as

$$\hat{S}_{pk} = \frac{1}{3} \Phi^{-1} \left[\frac{1}{2} \Phi \left(\frac{USL - \bar{x}}{s} \right) + \frac{1}{2} \Phi \left(\frac{\bar{x} - LSL}{s} \right) \right] = 1.2225$$

Finally, the practitioner decided that the submitted lot was accepted because $\hat{S}_{pk} = 1.2225 \geq k_N = 1.0968$.

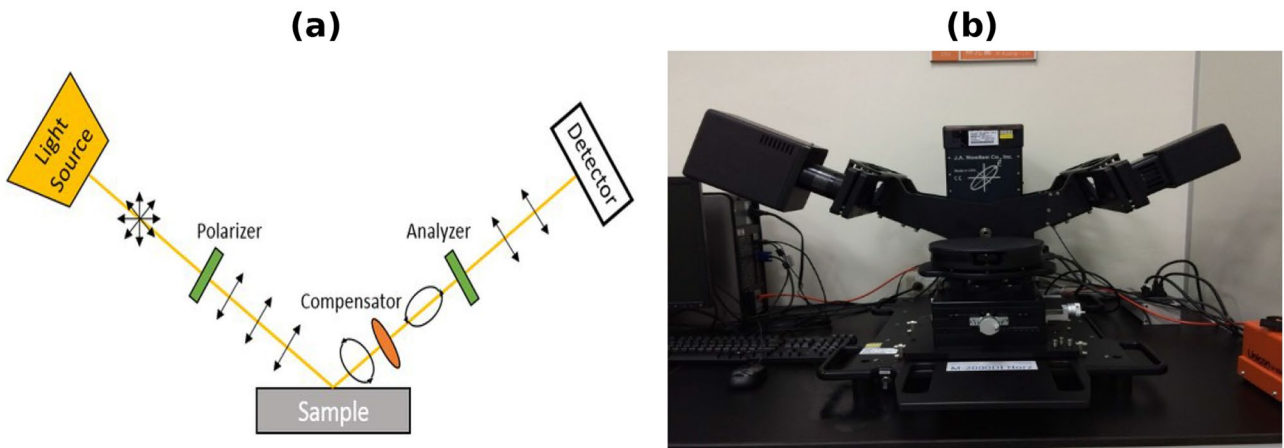


Fig. 8 (a) Schematic samples measurement through the spectroscopic ellipsometry [33]; (b) Experimental setup in the lab at National Cheng Kung University

Table 6 The measurement of these 59 items (unit: nm)

3.00	2.93	3.23	3.16	2.69	3.02	2.97	2.89	2.91	2.93
3.16	3.08	2.73	2.95	3.18	3.09	2.91	2.84	2.94	2.82
2.96	2.90	2.84	2.99	2.96	3.02	3.19	3.01	2.88	3.28
2.99	3.09	2.90	2.72	2.89	3.01	2.99	2.81	3.10	3.04
2.96	3.05	3.17	2.93	2.99	2.89	2.87	3.01	3.06	3.01
3.18	3.06	2.95	3.07	3.05	2.76	2.65	3.06	2.82	

$\bar{x} = 2.9753; s = 0.1342; p - \text{value} = 0.8174$

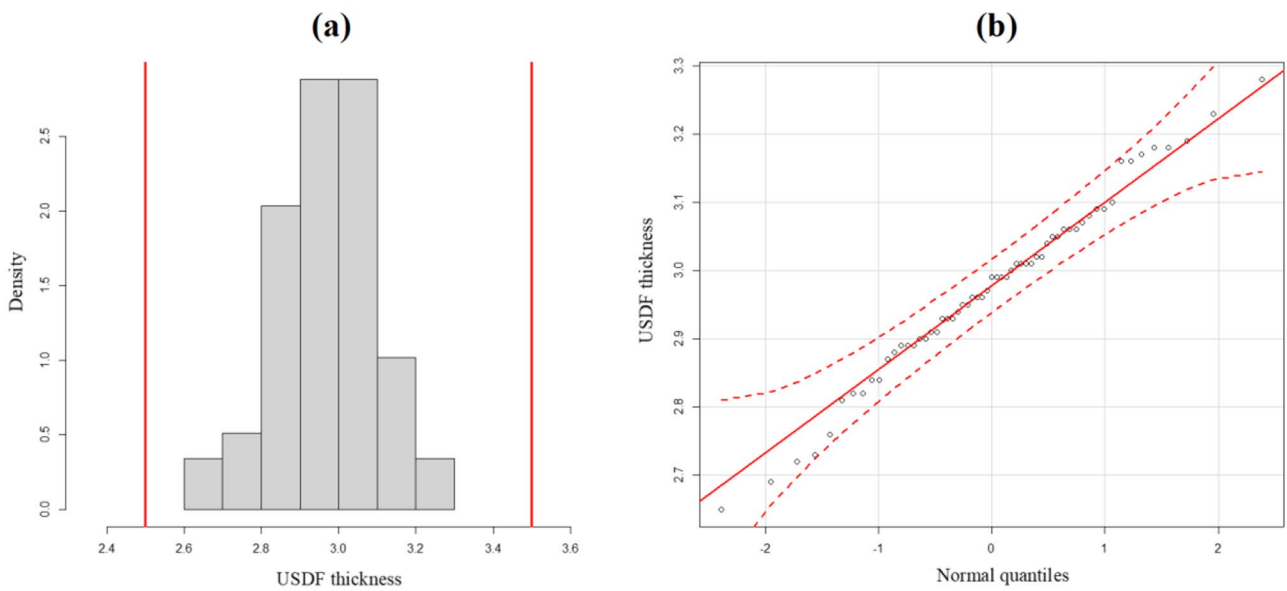


Fig. 9 (a) The histogram and (b) The normal Q-Q plot of the measurement

8 Conclusions

The varied sample size n and the lot-acceptance criterion k deployed in the NVSSP and TVSSP constitute the operational procedures of the S_{pk} -VQSS system in industrial distributions. They determine the efficiency of the inspection system and distinguish suppliers based on reliable standards and Six-Sigma process yield distributions. In this paper, we reviewed the existing S_{pk} -VQSS system, i.e., S_{pk} -VQSS-II system, and proposed two other types of S_{pk} -VQSS systems, i.e., the S_{pk} -VQSS-I system and S_{pk} -MVQSS system. Both S_{pk} -VQSS-I and S_{pk} -VQSS-II systems operate with a one-parameter changeover between normal and tightened inspection for the lot disposition, whereas the proposed S_{pk} -MVQSS system operates with a two-parameter changeover.

An R Shiny web app of the S_{pk} -VQSS-I, S_{pk} -VQSS-II, and S_{pk} -MVQSS systems was created to construct a convenient interactive UI for practitioners to determine their optimal system design. As a result of performance investigations in terms of ASN, OC curve, and ARL, their different strengths were discovered; the S_{pk} -MVQSS system has the lowest ASN, which can be seen as the most cost-efficient scheme, and the S_{pk} -VQSS-I system has the smallest α_{max} , β_{max} , and ARL values in response to process yield change, which can be seen as the most variation-responsive scheme. Although the existing S_{pk} -VQSS-II system neither dominates the sampling efficiency nor quickly responds to process variation, its straightforward administration in the NVSSP and TVSSP is suitable in the developmental stage of the supplier-buyer relationship.

A progressive lot-inspection process that integrates the S_{pk} -VQSS-I, S_{pk} -VQSS-II, and S_{pk} -MVQSS systems' strengths in different stages of the supplier-buyer trust relationship was conceptually developed. To filter out unreliable suppliers during the early stages of supplier-buyer relationships, we suggest adopting the S_{pk} -VQSS-I system that is sensitive to process yield changes. Subsequently, for those suppliers who are steadily developing the buyer's trust by their Six-Sigma process yield submissions, we recommend transferring the lot-disposition procedure to the S_{pk} -VQSS-II system for benefiting the simple administration and reducing the ASN-oriented inspection cost. Ultimately, for the suppliers who have demonstrated superior process-yield distribution, in the long run, we suggest employing the S_{pk} -MVQSS system that quickly adapts two-parameter changeover and maximizes the mutual benefit with the lowest ASN-driven cost of the inspection in order to establish lasting quality-sustainable supplier-buyer partnerships.

Lastly, the proposed S_{pk} -MVQSS system has a generalization mechanism that can be transformed into S_{pk} -VQSS-I and S_{pk} -VQSS-II systems when $k = k_N = k_T$ and $n = n_N = n_T$. This convertibility in the S_{pk} -VQSS systems makes the S_{pk} -MVQSS system a flexible and adaptive scheme.

Author contribution To-Cheng Wang contributed to the conceptualization, software, and the original draft. Jan-Yee Kung contributed to the case study and the editing. Bi-Min Hsu contributed to the literature overview and the practical implication. Ming-Hung Shu contributed to the organization, methodology, and validation.

Funding This work was partially supported by the Ministry of Science and Technology of Taiwan under grant numbers MOST 110-2222-E-013-001 and MOST 110-2221-E-992-086.

Declarations

Ethics approval Not applicable.

Consent to participate Not applicable.

Consent for publication The authors declare that this work has not been submitted elsewhere for publication, in whole or in part.

Conflict of interest The authors declare no competing interests.

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